1. What is the $x$-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ $\boxed{\text{(D) } -5}$ (E) $-10$

---

$y = \frac{1}{3}x^3 + 5x^2 + 24$

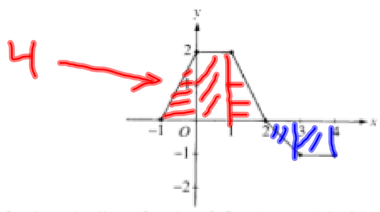
$y' = x^2 + 10x$

$y'' = 2x + 10$

$0 = 2x + 10$

$-10 = 2x$

$-5 = x$



2. The graph of a piecewise-linear function $f$, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 $\quad$ $\boxed{\text{(B) } 2.5}$ $\quad$ (C) 4 $\quad$ (D) 5.5 $\quad$ (E) 8

$\frac{1}{2}(1)(2) + (1)(2) + \frac{1}{2}(1)(2)$

$1 + 2 + 1$

$4$

$\frac{1}{2}(1)(1) + (1)(1)$

$\frac{1}{2} + 1$

$1\frac{1}{2}$

$4 - 1\frac{1}{2} = 2\frac{1}{2}$ or $\boxed{2.5}$

3. $\int_1^2 \frac{1}{x^2}\,dx =$ $\int_1^2 x^{-2}\,dx = \frac{x^{-1}}{-1}\Big|_1^2 = \frac{1}{-x^1}\Big|_1^2 = \frac{-1}{x}\Big|_1^2 =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\boxed{\frac{1}{2}}$ (D) 1 (E) $2\ln 2$

$\frac{-1}{2} + \frac{+1}{1} = -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$

4. If $f$ is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

slope

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

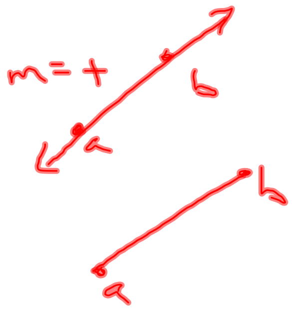
✓ (A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some $c$ such that $a < c < b$.

(B) $f'(c) = 0$ for some $c$ such that $a < c < b$.

✓ (C) $f$ has a minimum value on $a \le x \le b$.

✓ (D) $f$ has a maximum value on $a \le x \le b$.

✓ (E) $\int_a^b f(x)dx$ exists.



5. $\int_0^x \sin t \, dt = -\cos t \Big|_0^x$

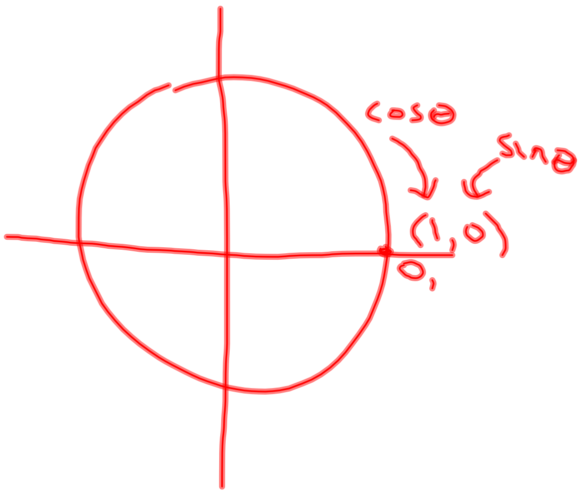
(A) $\sin x$  (B) $-\cos x$  (C) $\cos x$  (D) $\cos x - 1$  (E) $\boxed{1 - \cos x}$

$-\cos x - -\cos 0$

$-\cos x + \cos 0$

$-\cos x + 1$

$\boxed{1 - \cos x}$



$2^2 + 2y = 10$

$4 + 2y = 10$

$2y = 6$

$y = 3$

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $\boxed{-\frac{7}{2}}$ (B) $-2$ (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

$2x + 1 \cdot y + y' \cdot x = 0$

$2(2) + 3 + 2y' = 0$

$7 + 2y' = 0$

$2y' = -7$

$\boxed{y' = -\frac{7}{2}}$

7. $\int_1^e \left(\frac{x^2-1}{x}\right) dx =$ $\int_1^e \frac{x^2}{x} - \frac{1}{x}\, dx = \int_1^e x - \frac{1}{x}\, dx$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\boxed{\frac{e^2}{2} - \frac{3}{2}}$

$\frac{x^2}{2} - \ln|x| \Big|_1^e$

$\left(\frac{e^2}{2} - \ln e\right) - \left(\frac{1^2}{2} - \ln(1)\right)$

$\frac{e^2}{2} - 1 - \left(\frac{1}{2} - 0\right)$

$\boxed{\frac{e^2}{2} - \frac{3}{2}}$

8. Let $f$ and $g$ be differentiable functions with the following properties:

(i) $g(x) > 0$ for all $x$
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ ~~(B) $g(x)$~~ ~~(C) $e^x$~~ ~~(D) 0~~ $\boxed{\text{(E) } 1}$

$$h(x) = g(x) \cdot g(x)$$

$$h(x) = [g(x)]^2$$

$$h'(x) \quad 2(g(x)) \cdot g'(x)$$

$$h(x) = e^x g(x)$$

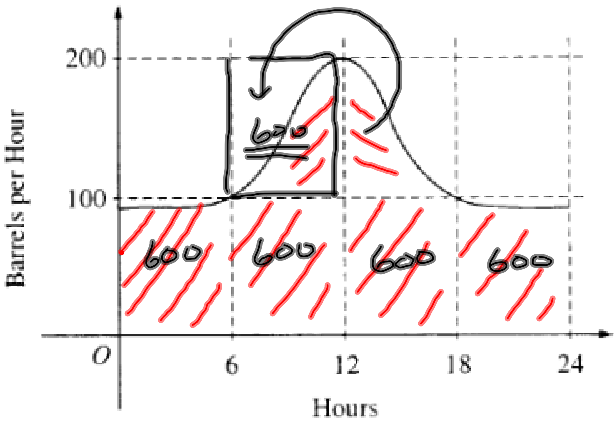
$$h'(x) = e^x g(x) + g'(x) \cdot e^x$$

$$= e^x g(x) + e^x g'x$$

$$h(x) = 1 \cdot g(x)$$

$$h'(x) = 0 \cdot g(x) + g'(x) \cdot 1$$

$$= 1 \cdot g'(x)$$




5 · 600

3000

9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

10. What is the instantaneous rate of change at $x = 2$ of the function $f$ given by $f(x) = \frac{x^2 - 2}{x - 1}$ ?

(A) $-2$  (B) $\frac{1}{6}$  (C) $\frac{1}{2}$  (D) 2  (E) 6

---

$$f(x) = \frac{x^2 - 2}{x - 1}$$

$$f'(x) = \frac{2x(x-1) - 1(x^2-2)}{(x-1)^2}$$

$$f'(2) = \frac{2(2)(2-1) - 1(2^2-2)}{(2-1)^2}$$

$$= \frac{4 \cdot 1 - 2}{1} = 4 - 2 = \boxed{2}$$

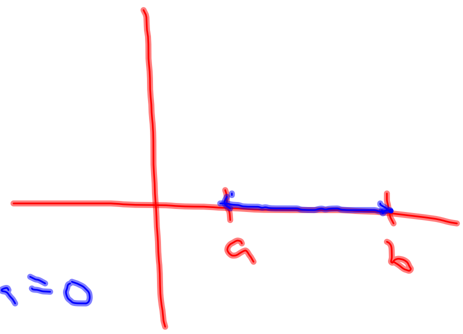
11. If $f$ is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$ $\underline{\underline{0}}$

$\boxed{\text{(A)}\quad 0}$ (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

$y = 2x + 6$

$y' = 2$

$\boxed{y'' = \underline{\underline{0}}}$




Area = 0

12. If $f(x) = \begin{cases} \ln x & \text{for } \boxed{0 < x \le 2} \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

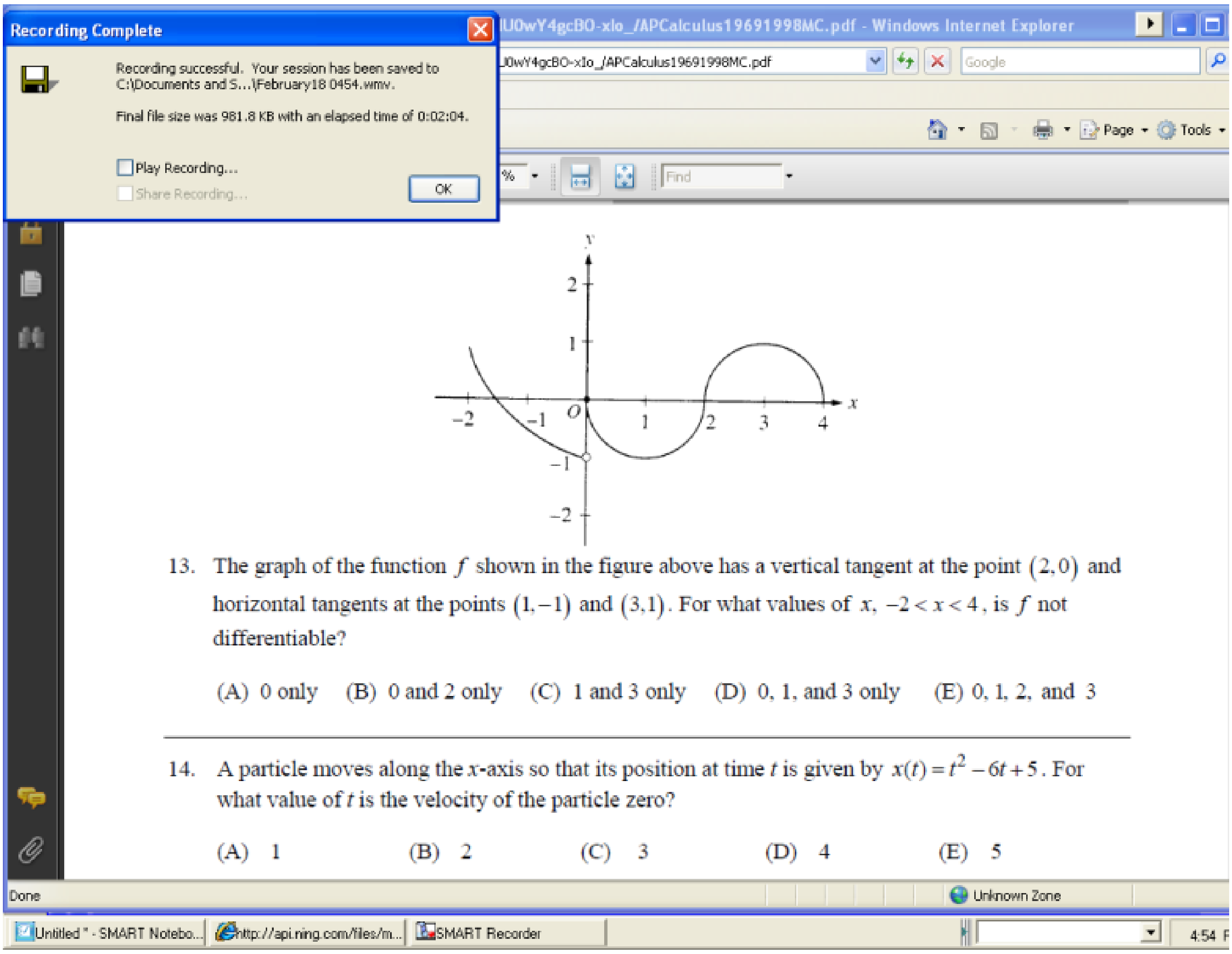
(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 $\boxed{\text{(E) nonexistent}}$

$\ln 2 \longrightarrow \qquad \longleftarrow 2^2 \ln 2$

$\ln 2 \qquad 4 \ln 2$

$\ln 2^4$

$\ln 2 \neq \ln 16$

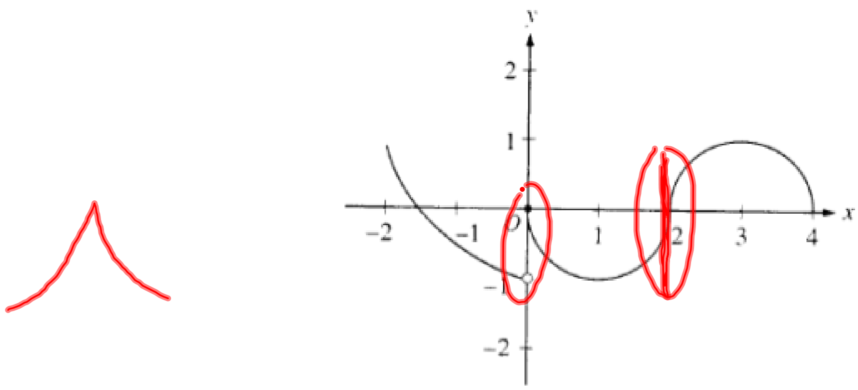



13. The graph of the function $f$ shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of $x$, $-2 < x < 4$, is $f$ not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14. A particle moves along the $x$-axis so that its position at time $t$ is given by $x(t) = t^2 - 6t + 5$. For what value of $t$ is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5



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(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$$x(t) = t^2 - 6t + 5$$

Velocity: $x'(t) = 2t - 6$

$$0 = 2t - 6$$

$$6 = 2t$$

$$\boxed{3 = t}$$

15. If $F(x) = \int_0^x \boxed{\sqrt{t^3+1}}\,dt$, then $F'(2) =$ $\sqrt{2^3+1} = \sqrt{8+1} = \sqrt{9} = 3$

(A) $-3$ (B) $-2$ (C) $2$ $\boxed{\text{(D) } 3}$ (E) $18$

16. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$ $\cos(e^{-x}) \cdot e^{-x} \cdot -1$

$= -e^{-x} \cos(e^{-x})$

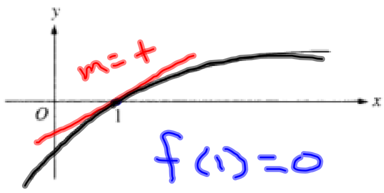
(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$




$f(1)=0$ $f'(1)=+$ $f''(1)= -$

17. The graph of a twice-differentiable function $f$ is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $\boxed{f''(1) < f(1) < f'(1)}$

(E) $f''(1) < f'(1) < f(1)$

$-$ $0$ $+$

$\boxed{f''(1) < f(1) < f'(1)}$

$x_1$ $y_1$

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

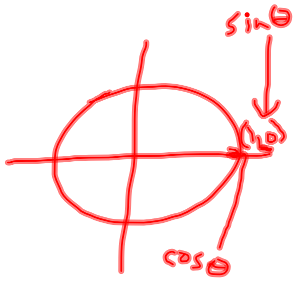
(A) $y = 2x + 1$ $\boxed{\text{(B)}\quad y = x + 1}$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x - 0)$$

$$y - 1 = x$$

$$\boxed{y = x + 1}$$



19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of $f$ has inflection points when $x =$

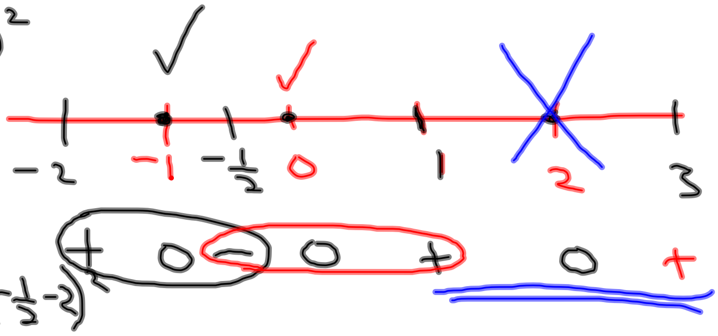
(A) $-1$ only (B) 2 only $\boxed{\text{(C) } -1 \text{ and } 0 \text{ only}}$ (D) $-1$ and 2 only (E) $-1$, 0, and 2 only

$f''(x) = x(x+1)(x-2)^2$

$= -2(-2+1)(-2-2)^2$

$= - \cdot - \cdot +$

$= +$




$f''(-\frac{1}{2}) = -\frac{1}{2}(-\frac{1}{2}+1)(-\frac{1}{2}-2)^2$

$= - \cdot + \cdot +$

$= -$

$f'(1) = 1(1+1)(1-2)^2$

$= + \cdot + \cdot +$

$= +$

$f''(3) = 3(3+1)(3-2)^2$

$= + \cdot + \cdot +$

$= +$

20. What are all values of $k$ for which $\int_{-3}^{k} x^2 dx = 0$?

(A) $-3$ (B) $0$ (C) $3$ (D) $-3$ and $3$ (E) $-3$, $0$, and $3$

$$\left.\frac{x^3}{3}\right|_{-3}^{k} = 0$$

$$\frac{k^3}{3} - \frac{(-3)^3}{3} = 0$$

$$\frac{k^3}{3} - \frac{-27}{3} = 0$$

$$k^3 + 27 = 0$$

$$k^3 = -27$$

$$\boxed{k = -3}$$

21. If $\frac{dy}{dt} = ky$ and $k$ is a nonzero constant, then $y$ could be

(A) $2e^{kty}$ (B) $\boxed{2e^{kt}}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

$y = 2e^{kt}$

$y' = 2ke^{kt}$

$\frac{dy}{dt} = ky$

$2ke^{kt} = ky$

$\boxed{2e^{kt}} = y$

22. The function $f$ is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is $f$ increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

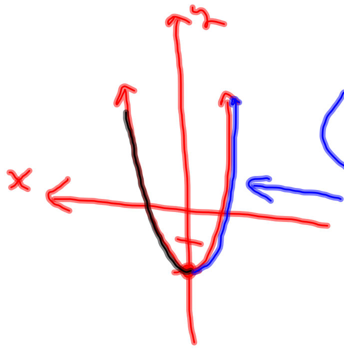
(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f'(x) = 4x^3 + 2x$

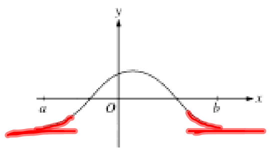
$0 = 2x(2x^2+1)$

$0 = 2x \qquad 0 = \cancel{\boxed{2x^2+1=0}}$

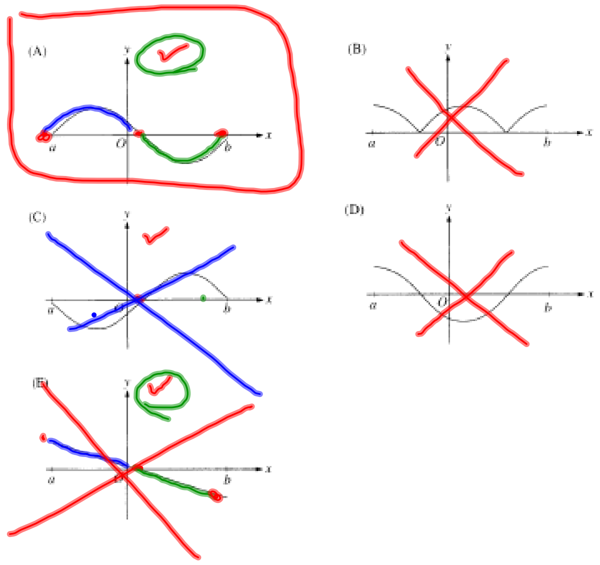
$0 = x$




$(0, \infty)$



23. The graph of $f$ is shown in the figure above. Which of the following could be the graph of the derivative of $f$?



24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

$a(t) = v'(t) = 3t^2 - 6t + 12$

$a(3) = v'(3) = 3(3)^2 - 6(3) + 12$

$= 27 - 18 + 12$

$= 9 + 12$

$= 21$

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\boxed{\frac{14}{3}}$ (E) $\frac{16}{3}$

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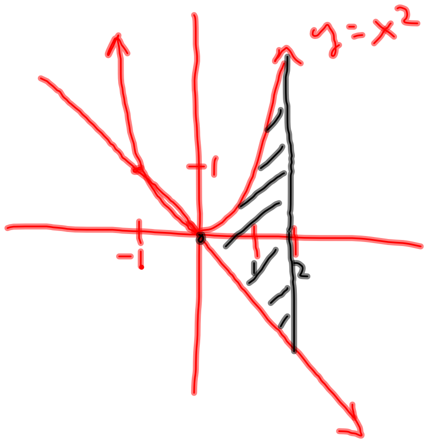



$$\int_0^2 x^2 - (-x)\,dx$$

$$\int_0^2 x^2 + x\,dx$$

$$\frac{x^3}{3} + \frac{x^2}{2}\Big|_0^2$$

$$\left(\frac{2^3}{3} + \frac{2^2}{2}\right) - \left(\frac{0^3}{3} - \frac{0^2}{2}\right)$$

$$\frac{8}{3} + 2 - 0$$

$$\frac{8}{3} + \frac{6}{3}$$

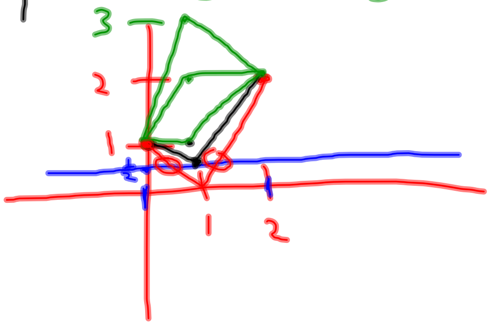
$$\boxed{\frac{14}{3}}$$

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | $k$ | 2 |

26. The function $f$ is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

(A) $\boxed{0}$  (B) $\frac{1}{2}$  (C) 1  (D) 2  (E) 3

1  0  0  0



$[a, b]$

27. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

$\boxed{\text{(A) } \frac{26}{9}}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx$$

$$= \frac{1}{2}\int_0^2 x^2 (x^3+1)^{1/2}\,dx$$

$$\to \cancel{\frac{1}{2}} \cdot \cancel{x^2} \frac{2(x^3+1)^{3/2}}{3 \cdot 3\cancel{x^2}} \Bigg|_0^2$$

$$\frac{(x^3+1)^{3/2}}{9}\Bigg|_0^2$$

$$\frac{(2^3+1)^{3/2}}{9} - \frac{(0^3+1)^{3/2}}{9}$$

$$\frac{\sqrt{9}^3}{9} - \frac{\sqrt{1}^3}{9}$$

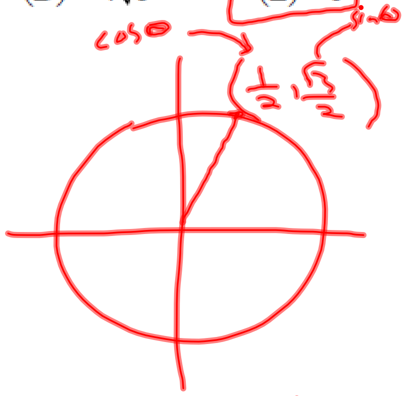
$$\frac{3^3}{9} - \frac{1}{9}$$

$$\frac{27}{9} - \frac{1}{9}$$

$$\boxed{\frac{26}{9}}$$

28. If $f(x) = \tan(2x)$, then $f'\left(\dfrac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $4$ (D) $4\sqrt{3}$ (E) $8$




$$f'(x) = 2\sec^2(2x)$$

$$f'\left(\tfrac{\pi}{6}\right) = 2\sec^2\left(2\cdot\tfrac{\pi}{6}\right)$$

$$= 2\sec^2(\pi/3)$$

$$= \frac{2}{\cos^2(\pi/3)} = \frac{2}{\left(\frac{1}{2}\right)^2} = \frac{2}{\frac{1}{4}} = 2\cdot 4 = \boxed{8}$$